Problem 12-20

The velocity of a particle traveling along a straight line is $v = (3t^2 - 6t)$ ft/s, where t is in seconds. If $s = 4$ ft when $t = 0$, determine the position of the particle when $t = 4$ s. What is the total distance traveled during the time interval $t = 0$ to $t = 4$ s? Also, what is the acceleration when $t = 2$ s?

Solution

The acceleration can be obtained by differentiating the velocity.

$$\begin{aligned} a &= \frac{dv}{dt} \\ &= \frac{d}{dt}(3t^2 - 6t) \\ &= 6t - 6 \end{aligned}$$

Therefore, after 2 seconds the acceleration is

$$a(2) = 6(2) - 6 = 6 \ \frac{\text{ft}}{\text{s}^2}.$$

The total distance travelled in the first four seconds is found by integrating the speed from $t = 0$ to $t = 4$.

$$\begin{aligned} s_T &= \int_0^4 |v(t)| \, dt \\ &= \int_0^4 |3t^2 - 6t| \, dt \\ &= \int_0^2 (-3t^2 + 6t) \, dt + \int_2^4 (3t^2 - 6t) \, dt \\ &= (-t^3 + 3t^2)\Big|_0^2 + (t^3 - 3t^2)\Big|_2^4 \\ &= [-2^3 + 3(2)^2] + [4^3 - 3(4)^2 - 2^3 + 3(2)^2] \\ &= 24 \text{ ft} \end{aligned}$$

The velocity is related to position by

$$v = \frac{ds}{dt} = 3t^2 - 6t.$$

Integrate both sides with respect to t to get $s(t)$.

$$s(t) = \int (3t^2 - 6t) \, dt = t^3 - 3t^2 + C$$

Use the fact that $s = 4$ when $t = 0$ to determine C. Then evaluate $s(t)$ at $t = 4$.

$$s(0) = C = 4 \quad \Rightarrow \quad s(t) = t^3 - 3t^2 + 4 \quad \rightarrow \quad s(4) = 20 \text{ ft}$$